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THE DYNEVAL (DYNAMIC ECONOMIC VALUES) MODEL
A GENERALIZED OVERVIEW

AC00C104 - Volume I

prepared for:

UNITED STATES ARMS CONTROL AND DISARMAMENT AGENCY

prepared by:

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Arlington, Virginia 22209

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ABSTRACT

This report summarizes performance by Decision-Science Applications, Inc., under the terms of contract no. AC00C104, "Completion of the Dynamic Economic Recovery Model," 14 April 1980.

The report includes three volumes. The first volume, titled "The DYNEVAL Model: A Generalized Overview," summarizes the basic design structure of the model; including the constraints; the economic objective function; the optimization methodology; and the basic mathematical formulation.

The second volume, titled "The DYNEVAL Model Documentation," includes a set of instructions for the use of ACDA personnel in operating the DSA economic model, as well as a detailed explanation of the mathematical structure of the model.

The third volume, titled "Economic Recovery Analyses," includes an illustrative analysis of Soviet economic recovery potential in three different attack scenarios, along with a set of sensitivity analyses which address the questions of level of aggregation (number of sectors), demand elasticity, and capital gestation periods. This volume also includes a brief analysis of the impact of increased military expenditures on the Soviet economy.

CONTENTS

FIGURES

1.0 INTRODUCTION

DYNEVAL is a national economic model in which the underlying decision processes are governed by a dynamic balance of value considerations as these considerations are reflected in the power structure of the economy. Whereas most national economic models are designed to project short term trends or business cycles in the normal operation of an economy, DYNEVAL is designed to provide a fundamental assessment of the effect of major economic disruptions on the operation of an economy. Given a major disruption (such as a drastic change in the level of defense expenditures, a cut-off in oil imports, the loss of foreign markets, or the destruction of production facilities), the model shows how capital investment and production resources can be allocated within the economy to mitigate the resulting economic damage; and it provides an assessment of the overall economic impact, assuming an efficient allocation of resources within the economy.

The model can be used in an essentially comparable way for either the U. S. or the Soviet economy, and it could be similarly applied to any economy for which corresponding input/output data are available. The basic economic data used by the model are derived from the input/output tables that are maintained by the U. S. Department of Commerce for both the United States and the Soviet Union. The model is equipped with a data input system which allows the user to specify any desired aggregation of the basic economic data. For example, the 468 sector representation of the U. S. economic data and the 88 sector representation of the Soviet economy can both be aggregated into a common set of sectors that might have 30 or 60 sectors for each economy. This allows the user to specify a greater level of detail in those economic sectors that are most pertinent to his particular analytical objectives.

DYNEVAL is designed as a generalized activity model which includes the dynamic time dependent effects of capital investment. For any specified scenario of economic disruption, the model will generate trajectories of economic activity which are as efficient as possible--relative to a dynamic economic objective function, which can be calibrated to explain the normal operation of the economy. Since the model generates efficient (or near optimal) trajectories, it is not useful in predicting specific imperfections in the operation of the economy, such as business cycles, which involve oscillations around an equilibrium economic state. Rather, the model is designed as a fundamental analysis tool for assessing either the theoretical adaptability of the economy to short-term disruptions; or the long-term responses of an economy to fundamental changes in technology, resource costs, and economic objectives.

The decision processes that govern the behavior of any large economic system reflect a dynamic balance between conflicting value considerations; between buyer and seller, between personal and social value criteria, and between short-term and long-term objectives. The pertinent value considerations which govern national economic behavior are not limited to the traditional considerations of profit and loss or personal gain. Real world economic behavior is inevitably influenced not only by the formal economic pressures of taxes, tariffs, and subsidies; but also by social and political forces which reflect the ideals and the ideology of the society.

The balance of economic priorities can be very different in different societies, so the actual balance cannot be derived from a priori or theoretical considerations. The priorities that actually operate in any society can be most accurately evaluated through a systematic analysis of the pattern of decisions as they occur in the normal operation of the economy. The economic projections of DYNEVAL are governed by a dynamic value logic which: is calibrated to reflect the observed balance of national economic priorities; is sensitive to current and anticipated future supply and demand relationships; and which accurately balances (or arbitrages) long-term versus short-term objectives.

DYNEVAL was originally developed as an analytical tool for assessing the potential for economic recovery in the aftermath of a large nuclear exchange. In the context of such massive economic disruption, it had proved difficult to apply standard econometric models, because the statistical correlations observed in the normal economy will not in general lead to reasonable behavior in the context of such a post attack economic environment. Because of DYNEVAL's ability to generate economic trajectories that are both reasonable and efficient even in the context of such massive economic disruptions, the model also has an obvious applicability in analyzing the potential capability of an economy to adapt to many other forms of disruption.

The model has been used in support of the Defense Nuclear Agency to develop insight with regard to priorities for industrial targeting. It is currently being used in support of the Department of Defense (Office of Net Assessment) to explore possible responses of the Soviet economy to changes in the available workforce and in the availability of oil and natural gas. The model has also been used to provide an assessment of the relative capability of the U. S. as opposed to the Soviet economy to support major increases in the level of defense expenditures.

This volume provides a general overview of the design and operation of the DYNEVAL model.

2.0 QUALITATIVE OVERVIEW OF THE MODEL

This section is designed to familiarize the reader with the basic design concepts of DYNEVAL without discussing the detailed mathematical formulation. Section 3.0 provides a compact (but somewhat simplified) introduction to the basic economic equations that are solved by DYNEVAL. For a discussion of the exact equations, the reader is referred to Volume II, DYNEVAL Documentation.

2.1 PURPOSE

As noted earlier, DYNEVAL is designed as a fundamental analytical tool for exploring potential economic responses to major economic trends and disturbances such as:

- Economic recovery after nuclear war.
- Effects of population and energy trends on economic growth.
- The vulnerability of an economy to interruptions of trade (such as might arise in the context of economic warfare initiatives).
- Response to major changes in the level or structure of military spending.
- Response to major structural changes in the pattern of taxes and subsidies.

In the context of such fundamental changes or disruptions within an economy, the DYNEVAL model can provide insight with regard to:

1. The potential adaptability of an economy;
2. Possible implications with regard to wage and price levels; and
3. The choice of an efficient economic strategy for coping with the disruption.

The DYNEVAL model is designed to predict the theoretical capability of an economy to respond to disruptions, assuming a reasonably efficient coordination and management of the national economy. The model does not attempt to predict how efficient or inefficient the national economic management will be in responding to any specific disruption; thus, as noted earlier, the model is not useful in predicting economic cycles or oscillations of the economy.

2.2 STRUCTURE OF MODEL

DYNEVAL is designed to generate a multi-year economic trajectory which balances supply and demand not only with regard to production and consumption within each time period, but also between time periods, with regard to the investment cost and discounted future value of production capital.

The optimal trajectory is calculated through an optimal control methodology using generalized lagrange multiplier optimization methods. Fig. 2-1 illustrates the basic structure of the model in the form of a three dimensional constraint matrix. Successive time periods are illustrated in this figure as a sequence of two dimensional activity matrices (such as the foremost matrix shown in the figure). Each row in this activity matrix corresponds to a specific economic commodity (or resource R_j)--such as coal, finished steel, or food products--that is produced and consumed within the economy. Each column in the figure corresponds to a specific activity--such as coal mining, manufacturing of steel products, or consumption of food--which contributes to the production and/or consumption of the economic commodities. The individual matrix elements represent the amount of each commodity (row) that is required per unit of activity in each column.

As illustrated in the figure, the model includes three basic types of activity: (a) production, (b capital investment,

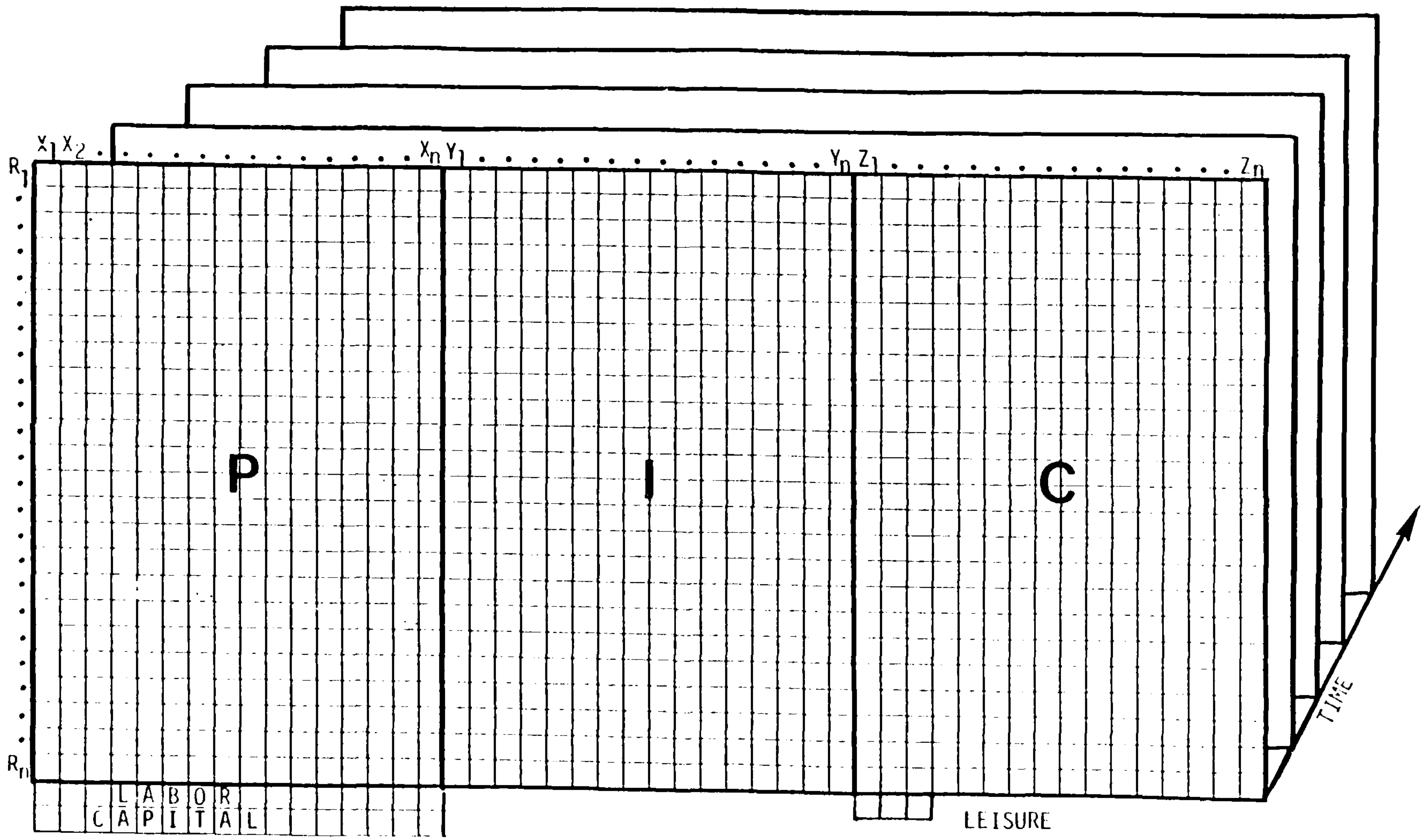

Figure 2-1. Illustrating the Basic Constraint Structure of DYNEVAL

and (c) consumption. The portion of the matrix concerned with each type of activity is designated in the figure by the symbols "P", "I", and "C". (An extension of the model is under development which will also include a detailed foreign trade sector, but this is not included in the present version of the model). The level of activity in each column is illustrated in the figure by a set of variables X_i, for the production activities, Y_i for the capital investment activities and Z_i for the consumption activities. These levels of activity are automatically optimized for each year to provide an economic trajectory which is not only in static equilibrium, with regard to production and consumption within each time period; but which is also in dynamic equilibrium, with regard to capital investment between time periods.

As a by-product of this optimization methodology, the model also calculates the shadow value of each commodity in each time period. These "shadow values" which, in a free economy would correspond very closely to prices, can be viewed more generally as the marginal value of an additional unit of each resource relative to the balance of economic objectives for the economy of interest. For clarity in the discussion, we will distinguish between different types of "shadow values" that occur in the model by using the value terminology of a free market economy. For example, we will speak of prices, costs, purchase costs for capital equipment--and the rental value, or rental cost, of manufacturing equipment. However, it is important to bear in mind that in a controlled economy the official price structure may be quite different than the calculated "shadow values". In the analysis of a controlled economy, the shadow prices provide fundamental information concerning the relative value of resources to the economy, but they may not correspond to the controlled prices within the economy.

Typically, the user of the model will be interested in the capacity of an economy to respond to various forms of economic disruption. The disruptions of interest could be of almost any form: ranging from immediate short term disasters such as the industrial damage that might result from a nuclear war, or changes in the international price of oil; to long term changes in the availability of labor; or the productivity of agriculture. For any such specified economic scenario, the model will estimate the most efficient strategy for coping with the specified disruptions; and it will generate an optimal economic response, in the form of a time dependent specification of the level of activity for each production, consumption and capital investment activity.

In order to calculate such an optimal economic response it is necessary, of course, to define both the economic objectives that are to be optimized and the physical constraints that will limit productive capacity. The DYNEVAL model uses conventional and well established mathematical forms to represent both the production constraints and the economic objectives. The model, however, differs from previous efforts to build optimizing economic models in two major respects:

1. A satisfactory and efficient convergence algorithm has been developed which provides reliable convergence to near optimal solutions even for very complex multi-year optimizations.
2. The economic objective function can be <u>calibrated</u> to realistically reflect observed economic priorities within the economy of interest.

To describe the functional design of the model we will need to discuss the following subjects:

- The Production Constraints
- The Economic Objectives
- The Optimization Approach

These issues are discussed briefly in the following sections.

2.3 THE CONSTRAINTS

2.3.1 The Production Function

The level of activity X_i of each production sector, i, is limited by: the availability of raw materials; the production capital, K_i, which is available within the sector; and the labor, L_i, that is allocated to the sector. The raw material constraints are represented in the model in the form of conservation requirements for each resource. Specifically, the total consumption of each economic commodity (including consumption by other production processes, consumption in capital investment, final demand consumption, and net imports) cannot exceed the total production. Thus, the total production of each such commodity defines a feasibility constraint on the combined levels of activity for all activities that use the commodity.

The maximum feasible production level X_i for each production activity also depends on the availability of capital and labor. At the beginning of each time period, the inventory of production capital available for each production activity is calculated, taking into account the effects of depreciation and capital investment in previous time periods. The resulting inventory of production capital that is available for production within any time period is, therefore, completely determined by previous decisions and cannot be changed by decisions made within the time period.

The actual level of production in each economic sector can nevertheless be adjusted within limits by changing the allocation of labor. For example, in many industries there is substantial additional productive capacity that can be exploited by using overtime labor or by increasing the number of shifts of labor that are employed at the plant. Similarly, if there is excess capacity in any industrial sector, the level of production can be reduced by closing factories, by laying off workers, or by shortening the work week. Thus the actual level of production in each economic sector can be adjusted within a time period by changing the allocation of labor.

The model uses a conventional translog production function, $P_i\ (K_i,\ L_i)$, to specify the relationship between the available production capital, K_i, the applied labor, L_i, and the level of production, X_i, that can be achieved in each production sector, i. Specifically, the production X_i cannot exceed the feasible producivity $P_i(K_i,\ L_i)$, or

$$X_i \leq P_i(K_i, L_i) \qquad (1)$$

Because leisure time is assumed to have a positive value, the optimal solutions will never use more labor than is needed to achieve the desired production level (so Eq. 1 above could just as well be written with an equality sign in place of the less-than-or-equal symbol that is shown).

The translog production function defines a parametric family of production functions that can be automatically calibrated to match the economic data for any production sector. The general mathematical form of the translog production function is shown below.

$$P_i\ (K_i, L_i) = G_i \left[f_i K_i^{-B_i} + (1 - f_i) L_i^{-B_i} \right]^{-(1/B_i)} \quad (2)$$

The variable, B_i, is a user controlled parameter that is used to specify the assumed elasticity of the trade-off between labor and capital in each industry i.

For any specified choice of the elasticity, B_i, the model will automatically calculate the specific values for the other defining parameters (specifically f_i, and g_i) that are required for consistency with the actual economic data. Through the use of these parameters the model automatically generates a specific form, P_i, of the translog which is compatible with a stated equilibrium in the baseline economy. (Specifically, given the actual level of labor, L_i, and the production capital, K_i, in the baseline data, the model calculates values of f_i, and G_i that are consistent with: the relative value of labor, production capital, and the actual production level).

Because the elasticity factors, B_i do not have a simple intuitive meaning, most users will need assistance in selecting appropriate values of B for specific industries. The model, therefore, allows the user to specify the value of B indirectly, by providing an estimate of the maximum amount by which production could be increased by simply increasing the applied labor.

Experience in World War II demonstrated that, in many labor intensive industries that typically operate only one forty hour shift, productivity can be increased by a factor of almost 2.5 simply by increasing the level of applied labor. Efforts to increase the productivity of capital equipment beyond this level were rarely successful because of the requirements for maintenance and repair. Industries such as oil refining and oil extraction, that are highly mechanized and operate in a continuous production mode, are much less able to increase productivity by adding labor. For such industries the maximum factor by which productivity can be increased by adding labor is unlikely to be more than about 20 to 25%. The real estate industry, of course, provides an extreme example in which the productivity (available office and housing space) is determined almost entirely by the invested capital and is very insensitive to the applied labor (real estate salesmen). Fig. 2-2 shows some illustrative graphical examples of the resulting translog production functions that are generated by the model based on a user's specification of a maximum capital productivity in each sector.

2.3.2 The Inventory of Production Capital

As noted earlier, the capacity for production in any economic sector is limited both by the allocated labor and by the inventory of production capital. To allow for the time required to construct new production facilities, the model provides a time delay (or gestation time) between the time period when the capital investment is made and the time when it becomes available to the production activities. The inventory of production capital that is available for any time period is calculated by first depreciating the inventory that was available in the previous time period, and then adding any new production capital that has completed its gestation delay from the time capital investments were made in previous time periods. DYNEVAL permits each productive activity to use up to two types of capital, but each type of capital can only be used by the specific sector for which the capital investment was made.

Mathematically, capital investment activities within the model are treated simply as a bookeeping transaction, which transfers goods and services from the inventory of consumables to the inventory of production capital for a specific economic sector. This transfer takes place in two steps. In the first step, the goods and services are subtracted from the supply of consumables for the time period when the investment takes place. In the second step, after the specified gestation period, the invested capital becomes available to the intended production activity.

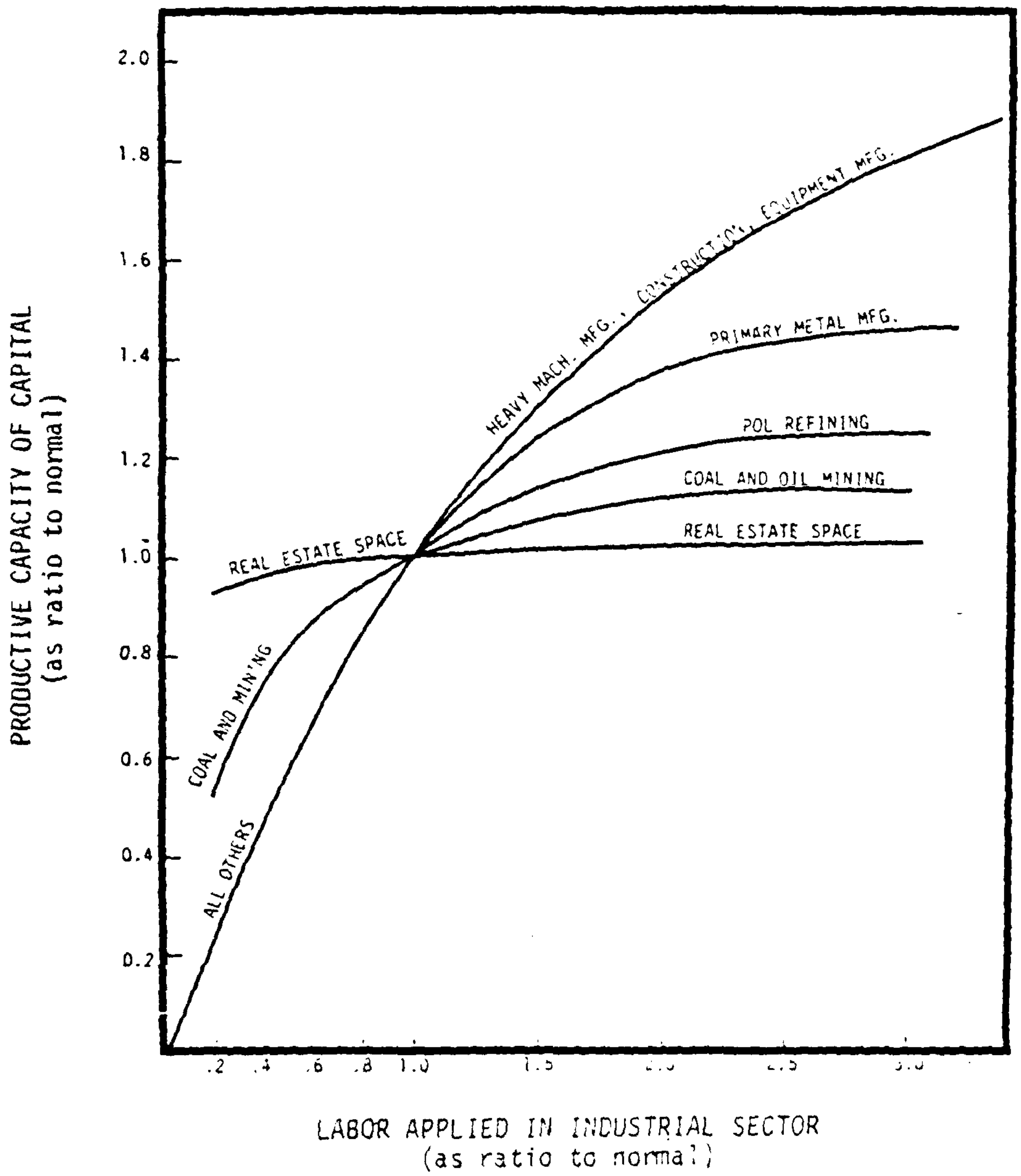

Figure 2-2 Illustrative Productivity of Capital as a Function of Labor Applied. (The figure shows curves calculated by the model using the translog production function, after the user specified a maximum capital productivity)

The model allows the user to specify the gestation time he feels is appropriate for each industrial sector. Any desired gestation time can be specified, as long as it is greater than one-half of a time period. (If it were desired to specify a gestation time less than six months, it would be necessary to run the model using time steps of less than one year).

When deciding on specific capital gestation times to use within the model, one should consider the _average_ gestation time for capital during the construction of a new facility rather than the _total_ construction time. Obviously, the capital expenditures that are made late in the construction process have a relatively short gestation time, whereas those that are made early have a relatively long gestation time. To estimate the average gestation time one should theoretically consider the total profile of capital expenditures in the construction process and then compute the average gestation time.

The specific goods and services (or economic commodities) required to provide one unit of production capital in each industry are defined by the portion of the direct requirements matrix labeled as "I" in Fig. 2-1. For the U.S. economy this data was obtained directly from the capital flow tables that are maintained by the Department of Commerce as a part of their input/ouput data. Since corresponding Soviet capital flow data is not available, the required data for the U.S.S.R. was estimated using the U.S. capital flow as a starting point. The actual numerical entries for the U.S.S.R., however, were adjusted to reflect both the differences in the prices of specific commodities in the U.S.S.R. and the differences in the level of production activity for the corresponding economic sectors.

The way the capital investment activities are treated in DYNEVAL, as a simple bookeeping transaction which does not require any labor, may seem strange at first to those who are not familiar with the way capital flow is normally represented in input/output data. The reason that it can be legitimately treated this way is that both the labor and the raw materials required for any new production facility are fully included in the goods and services specified for the capital investment. For example, the required construction labor is "purchased" indirectly through the construction sector of the economy.

2.4 THE ECONOMIC OBJECTIVE FUNCTION

2.4.1 Fundamental Concepts

The ultimate objective of a national economy is to provide goods and services that are required to: support the population; operate the government; provide for national defense; and to provide a certain amount of leisure for the laboring population. From this broad perspective, all of the production activities and all of the capital investment activities can be viewed simply as a means to an end. The production activities are required in order to provide the goods and services that are needed for consumption by the population, the government, the military, and for capital investment. The capital investment activities in turn, are necessary in order to maintain and expand the national productive capacity that is needed to meet future production requirements. Because both the production activities and the capital investment activities (the P and I portions of the matrix in Fig. 2-1) are only a means to the real economic objectives, they are not directly represented in the DYNEVAL economic objective function.

The DYNEVAL economic objective is defined as a function of the ultimate consumption activities that are supported by the economy. (In Fig. 2-1 these consumption activities are represented by the C portion of the matrix). Specific consumption activities that are typically represented in the DYNEVAL objective function include: consumption by the people of all commodities; consumption by the people of "leisure" (which must, of course, compete with the need for productive labor) consumption required for the national defense, and other consumption by national and state governments.

The relative importance assigned to these areas of economic consumption can be very different in different societies. For example, the balance that is reached between the availability of leisure and a high standard of living can be quite different depending on the degree to which the society has a strong work ethnic. The balance between national defense and personal consumption depends not only on the extent of real and perceived military threats, but also on the extent to which military objectives are influential in the control of the economic system. The balance among the various consumables can also vary depending on local preferences, traditions, and expectations within the society. Obviously, there is no way to derive the value criteria for any specific national economy on an a priori basis. However, the way that goods and services are actually distributed within an economy provides a baseline definition of actual economic demand within the economy that can be used to calibrate the objective function.

In order to properly calibrate effective objectives for a national economy, we need to consider not only the way objectives are balanced within an individual time period, but also the way resources are balanced between present and future economic objectives. For example, a society that places great importance on the future may make substantial sacrifices in the current standard of living in order to invest resources for a better future, whereas a society that is focused primarily on the present is less likely to make such sacrifices.

2.4.2 Estimation of Discount Rate for Future Value

To provide a realistic representation of national objectives with regard to the balance between present and future economic objectives, DYNEVAL provides for automatic calibration of an effective discount rate for future values that is required to "explain" the level of investment activity within the economy.

Theoretically, in an optimal economy, the "effective" discount rate for future values should be the same in all economic sectors. In practice, however, economic decisions within an economy are unlikely to be quite that rational. For various cultural or social reasons, certain sectors tend to receive more or less than an optimum level of capital investment. For example, in the U.S. economy the tax structure tends to favor investment in private homes and real estate, and it tends to discourage investment in production facilities. As a consequence, the economy operates as if the effective discount rate in the real estate sector is lower than in the manufacturing sectors. Specifically, the effective discount rate in the real estate sector is close to 4% per annum, whereas the effective discount rate for resources invested in manufacturing facilities tends to be closer to 10% or 12%.

In order to properly represent such systematic deviations from theoretically optimal behavior, DYNEVAL provides a separate discount factor that can be automatically calibrated to the actual investment behavior in each economic sector. Although the use of such sector dependent discount rates precludes theoretically optimal solutions, it permits a far more accurate representation of actual investment priorities within the economy. If a true optimum solution is required, the user can easily substitute an average universal discount rate.

2.4.3 Importance of Demand Elasticity

Under circumstances of major economic disruptions, there are likely to be critical shortages of specific economic resources. When this happens, the supply of the critical resources can be of great importance to the economy and the effective "unit value" of these resources to the economy can be many times their normal value.

This obvious dependence of the unit value of goods and services on the available supply is specified within the model in the form of a demand elasticity for each consumption activity. The less elastic the demand is assumed to be, the more rapidly the unit value will tend to escalate in the context of a supply scarcity.

In traditional economic models that are designed to deal with the performance of an economy in a near equilibrium environment, it is often possible to obtain reasonable results without explicitly representing this dependence of value on the available price. However, in a model that is designed to study the response of an economy to major economic disruptions, this familiar simplifying assumption is no longer tenable.

In order to make reasonable decisions about how to allocate critical resources between production activities, and how to balance the priorities for capital investment against current consumption, it is absolutely essential to have some way of estimating the relative importance of specific resources to the economy, both within the individual time periods and between time periods.

Experience with the model has shown that it is usually not necessary to have a very accurate estimate of actual demand elasticity for each consumable. For most problems, the model will give very reasonable results over a surprisingly wide range of demand elasticity. For example, in calculating rates of economic recovery following a large scale nuclear attack, the effective rate of recovery was found to be almost independent of the overall demand elasticity over a range of from .5 to 2.0. Thus, although it is essential to represent the dependence of value on price within the model, it is not essential to estimate this dependence with great accuracy.

2.4.4 Specific Demand Curves and the Economic Utility Function

Mathematically the optimization process in DYNEVAL can be interpreted *either* as the maximization of a utility function, or as the generation of an economic equilibrium solution (that satisfies the familiar equilibrium requirements for a balance--between cost and value, both within the individual time periods and between time periods.) Because of the essential equivalence between these two interpretations, it is appropriate to discuss the economic objective function *both* as a utility function *and* as a specified set of final demand elasticities.

The demand elasticity for each consumption activity in DYNEVAL is specified using the well known "constant elasticity" demand function. Using this generalized demand function, one can specify any desired elasticity ranging from maximum elasticity, where value is totally independent of supply, to a minimum elasticity, in which consumption is rigidly determined independent of cost.

Fig. 2-3 illustrates, in a qualitative form, the relationships between demand elasticity and the corresponding utility function. The curves at the top of the page illustrate the relationships that would obtain with the constant value simplification. This simplifying assumption implies a linear contribution U_i to the utility function, and a unit value V_i for the consumption activity that is completely independent of the level of consumption C_i. The curves in the middle of the page correspond to the widely used logarithmic utility function. This

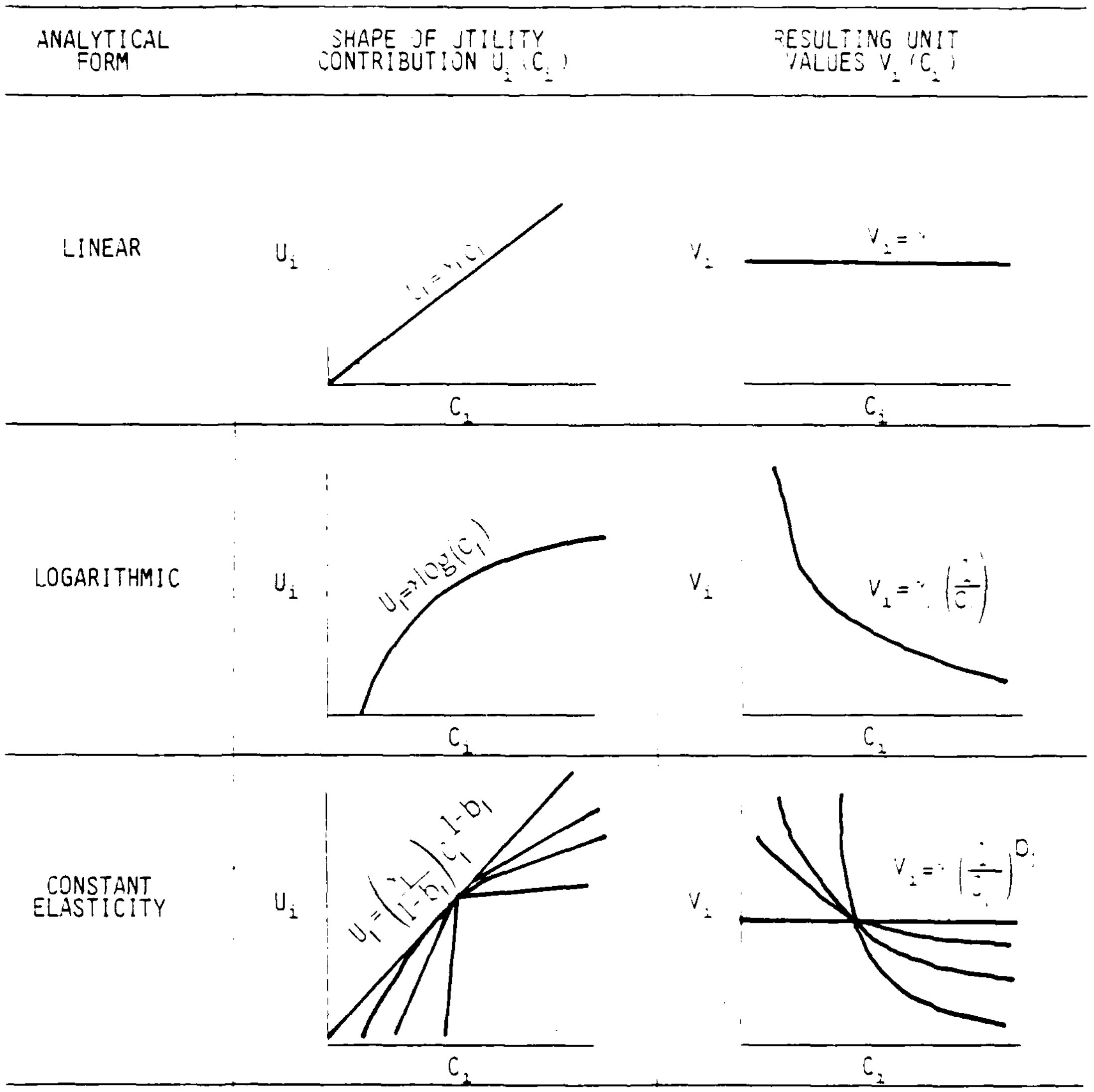

Figure 2-3. Illustrative Comparison of Alternative Demand Functions: Showing the Unit Value V_i as a Function of the Consumption Level C_i on the Right and the Corresponding Contribution to the Utility V_i as a Function of C_i on the Left.

utility function implies a demand elasticity of 1.0 (in which the unit value V_i is inversely proportional to the supply or consumption level C_i). The constant elasticity demand function, which is actually used in DYNEVAL, allows the user to specify <u>any</u> of a family of demand curves such as are illustrated in the lower set of curves. In these demand curves the unit value for each consumption actually is inversely proportional to the consumption level C_i raised to the power b_i. The corresponding contributions U_i to the utility function (as illustrated on the left) range from linear (for $b_i = 0$), through logarithmic (for $b_i = 1.0$) to an almost discontinuous step function (for $b_i \cong \infty$).

To provide even more flexibility in the specification of demand, DYNEVAL allows the user to specify <u>minimum</u> allowed levels of consumption for each of the consumption activities C_i. This minimum allowed level is incorporated into the demand and utility equations simply by substituting $(C_i - a_i)$ in place of C_i.

To avoid undue complexity, the overall DYNEVAL utility function is defined as a simple summation of the contributions from all consumption activities, i. Specifically, the overall utility U(t) within a time period t is defined simply as:

$$U(t) = \sum_i U_i\left(C_i(t)\right) \tag{1}$$

The overall utility function U that is optimized by DYNEVAL is simply the discounted value of this utility function summed (integrated) over the time periods. Specifically:

$$U = \sum_{t=1}^{t=\infty} U(t)e^{-\rho t}\Delta t \tag{2}$$

where ρ is the effective discount rate for future values, and Δt represents the chosen length of an individual time period. Obviously, when different discount rates are used in different industries, the optimization is not exact and the average "effective" discount rate is somewhat unclear. Nevertheless, even in such cases the basic concept of an approximate utility function is still relevant.

2.4.5 Demand Correlations and the Evaluation of Demand Elasticity

In any large economic system there are inevitable correlations in the actual demand function. For example, the use of rubber products (particularly tires) is closely correlated with the use of gasoline. The combined use of coal, oil, and electricity for space heating is closely related to the space that needs to be heated. Whereas the total consumption of food calories tends to be quite inelastic, there are relatively elastic trade-offs between substitutable nutrients such as meat and fish, or potatoes and grain.

Moreover, for most consumables it is important to distinguish between short term and long term elasticity. For example, in the short term, gasoline consumption is quite inelastic. In the long term the demand for gasoline is considerably more elastic because one can substitute more fuel efficient cars, provide better public transportation, and even revise residential patterns to minimize transportation requirements.

It is obvious that the development of a comprehensive demand elasticity matrix--one that could properly reflect such correlations in demand among all consumables, on both a short term and a long term basis--would require a large research effort. To avoid this massive data requirement and to maintain a simple intuitive representation of demand elasticity, DYNEVAL uses a simple additive utility function corresponding to the uncorrelated demand activities. Nevertheless, the DYNEVAL user is able to represent a wide variety of realistic assumptions concerning the correlations in demand by exercising care in the definition of consumption activities.

In the simplest and most routine representation of demand, consumption by the people of each commodity is treated as a separate consumption activity. Obviously, in this routine representation, the model assumes independence in the demand for all commodities. On the other hand, the user of the model can decide to include all or part of the consumption of selected commodities in specially defined consumption activities that are designed to generate the proper demand correlations. For example, one could define a special activity labeled "transportation" which would account for a large fraction of the consumption of both rubber and gasoline. Similarly, one could define a special activity labeled "shelter" which would account for a large fraction of the consumption of both real estate and heating oil.

Obviously, the user himself must decide to what extent it is necessary or desirable to introduce such extra consumption activities for any specific analysis. Once a decision has been made to do so, DYNEVAL's data aggregation subsystem can be used to facilitate the required data manipulations.

As should be apparent from the foregoing discussion, the DYNEVAL demand function assumes that the most important demand correlations can be modeled by a careful definition of the ultimate consumption activities. When the final demand activities are appropriately defined in terms of their ultimate functions the degree of correlation is much reduced. For example, the degree of correlation between broad functional elements such as shelter and transportation is likely to be far less than between individual commodities such as rubber products and gasoline.

In selecting demand elasticities to use in any specific study, one should take into account the degree to which the desired correlations have been represented in the specification of the consumption activities. For many applications it may be sufficient to use different elasticities for different consumables. For example, in most applications the short term inelasticity of fuel oil consumption could be represented simply by specifying a high inelasticity for fuel oil consumption, rather than by correlating fuel oil consumption with shelter usage.

When the consumption activities are defined in an appropriate functional form(so that they reflect the major functional correlations) there is reason to believe that the underlying demand elasticity for such fundamental consumption activities will be relatively close to 1.0 for almost all consumption activities.

The evidence for this can be developed in many ways. In terms of our common sense experience, we know that an increase in salary of any given percentage seems to be about equally welcome--almost regardless of one's actual salary level. If it were exactly true that a given percentage increase is equally important at all salary levels, one could rigorously derive the general logarithmic utility function, which corresponds to an elasticity of 1.0. The logarithmic utility function also seems roughly applicable to housing since a move from a two room house to a four room house seems to be just about as important a step as a move from a four room to an eight room house. Thus, it seems reasonable to use an overall elasticity of 1.0 as a starting point for the development of elasticity estimates for the most fundamental consumption activities. Obviously, to develop detailed estimates for any specific scenario, it is important to take into account some of the obvious functional correlations.

2.4.6 Elasticity of Leisure

In addition to the demand elasticity of commodities, the model also requires an estimate of the demand elasticity of leisure. There is a great deal of evidence which suggests that the demand elasticity of leisure is very close to that for commodities. Around the world, human societies operate at an extremely wide spectrum of standards of living. Nevertheless, the normal work week is remarkably close to forty hours in almost all societies. This suggests: first, that overall consumer demand is quite elastic and close to 1.0 over a wide range of levels of consumption; second, that the demand for leisure follows a law of diminishing returns which results in a work week close to forty hours over a wide range of levels of affluence.

Our tests using the model have indicated that these obvious characteristics of the real world consumption of leisure are rather accurately duplicated in the model by using an elasticity of 1.0 and by defining the maximum time available for work plus leisure in the baseline economy to be about twice the applied labor. In effect, this divides the total week of 168 hours into two almost equal parts as follows: 84 hours for sleep, eating, and other essential functions; and 84 hours that can be divided on a discretionary basis between labor and leisure.

Obviously, a really careful study might refine these estimates, but for most practical purposes, this method of defining the demand for leisure seems to operate in a generally satisfactory way.

2.5 THE OPTIMIZATION PROCESS

2.5.1 The Role of Optimization in DYNEVAL Decision Processes

The performance of any economic system is governed both by the physical constraints which limit the feasible production alternatives; and by an economic decision process which determines the choices that will be made among the feasible alternatives. The choices that are likely to be made will depend in turn on the balance of economic objectives that motivate the decisions. The previous three sections have discussed both the physical constraints and the representation of national economic objectives. This section provides a brief overview of the way decision processes are represented within the model.

All of the economic decisions in DYNEVAL are accomplished automatically by means of a mathematical optimization process. The specific economic decisions that are made in this way for each time period are as follows:

1. The allocation of worker's time between labor and leisure.
2. The allocation of labor among all producing activities.
3. The level of production in each economic sector.
4. The total capital investment in each time period.
5. The allocation of capital investment among the productive sectors.
6. The level of activity for each consumption activity.

Because all of the foregoing decisions are accomplished automatically by the optimization process, the model results are relatively independent of the detailed judgments by the analyst or programmer that are required by most models (either in the program itself, or in user selected parameters that determine the appropriate rates of capital investment). The very small number of user specified parameters required by the model makes it a much more convenient analytical tool for the comparison of alternatives, because the results are much less likely to be biased by arbitrary and hidden judgments of the analyst or programmer.

2.5.2 Design Concept for Optimization System

Fundamentally, the DYNEVAL decision processes are accomplished by an iterative lagrange multiplier optimization process. More specifically, the model uses two separate optimization systems:

1. A Single Time Period Optimizer
2. A Trajectory Optimizer

The single time period optimizer can be viewed as a subroutine that can be called by the trajectory optimizer to calculate an optimal mix of activities for a specific time period, given certain assumptions about the overall trajectory. It is the responsibility of the trajectory optimizer to link together the results provided by the single time period optimizer in such a way that the entire process converges to an overall optimum trajectory.

In order to calculate the optimal levels of activity within a time period it is necessary to have two types of information which are dependent on the performance of the economy in other time periods. First, it is necessary to know how much production capital is available for each economic sector. This, of course, depends on the decisions that were made in earlier time periods concerning capital investments for each sector. Second, to make logical decisions concerning the level of specific capital investment activities within the time period, it is necessary to know the extent to which additional production capacity will be needed in future time periods. Thus the optimization of a single time period requires a flow of information from previous time periods to specify the available production resources, and a flow of information from future time periods to establish valid priorities for investment in new production capital. Fig. 2-4 illustrates this basic flow of information between time periods as it is used during the trajectory optimization process.

As illustrated in the figure, the external information requirements for each time period consist of: (1) a quantity, K, which specifies the inventory of production resources for each economic sector; and (2) a quantity, λ, which represents the discounted value of new capital investments in each sector to future time periods. The information concerning the inventory of production capital K is projected <u>forward</u> in time from the early to the later time periods taking into account both the depreciation of existing production resources and the increments,

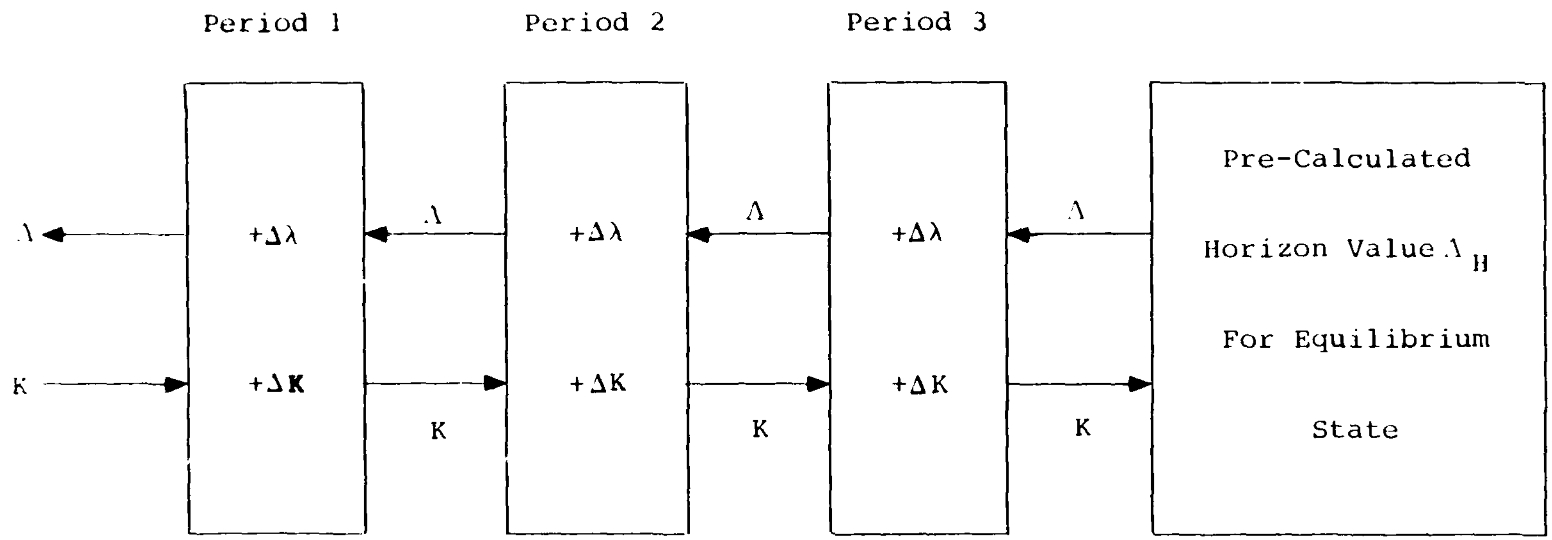

Figure 2-4. Illustrating Flow of Information Between Time Periods During the Trajectory Optimization (Although the figure shows only three time periods, typical runs of the model are done using 20 to 40 time periods to ensure convergence to the horizon equilibrium and avoid discontinuities in the interface of the Horizon State)

ΔK, of new capacity that are generated by capital investment activity in each time period. The information concerning the discounted future value, Λ, of new production capital is projected backward in time from the later to the earlier time periods, taking into account both the discounting of future values over time and the estimated marginal value, $\Delta\Lambda$, for additional production capital within each time period.

In order to make this kind of optimization process practical, there has to be some way to avoid processing unlimited numbers of time periods. The traditional way to do this is to define a horizon time, sufficiently far in the future that is beyond the period of interest, and to assign a specific horizon value Λ_H to represent the value of production resources delivered to the horizon. Obviously, it is desirable, if possible, to define this horizon value in such a way that it will not distort the decision processes in earlier time periods. In DYNEVAL this is accomplished by pre-calculating an ultimate equilibrium state for the economy that would apply after all of the transient effects of the economic scenario have damped out. The value Λ_H of production resources which is calculated for this long term equilibrium state is used to provide the horizon values for the dynamic calculation. In typical applications, the horizon is placed about twenty to forty time periods into the future so that it has no discernable effect on the decisions in earlier time periods.

2.5.3 Overview of Trajectory Optimizer

The development of an optimal trajectory is accomplished by an iterative computation method that proceeds roughly as follows. First, an initial feasible economic trajectory is generated. This is accomplished by making successive calls on the single time period optimizer, beginning with the first time period and moving one time period at a time to later time periods. To ensure feasibility, the production resources available to each successive time period are calculated to reflect both the depreciation of capital and the actual level of capital investment generated in the previous time periods. During this process the single time period optimizer calculates not only the capital investment ΔK for each economic sector, but also the marginal value $\Delta\lambda$ for additional production capital of each type within the time period.

Given this information on the marginal value of production capital in each time period it is possible to calculate exactly the discounted future value λ of production capital for trial trajectory that was generated. To calculate these values, the model starts at the horizon (where the value λ_H is known) and works backward one time period at a time. In each step the value is first increased by $\Delta\lambda$, to reflect the marginal value of capital equipment in the current time period, and it is then discounted (at the specified discount rate for that resource) to provide a new estimate λ' of the discounted value of capital investment for the preceding time period. These new computed values λ' are then utilized to refine the values of λ that are used in thte trajectory optimization. With these refined values of λ the system is ready to proceed with the next iteration.

The trajectory optimizer repeats this iterative process until it converges to a dynamic equilibrium, in which the new calculated values λ' are almost exactly equal to the estimated values λ that were used in the trajectory optimization. When this convergence process reaches a pre-determined level of accuracy the iteration process stops and the resulting optimal trajectory is printed out for study by the analyst.

2.5.4 Overview of Single Time Period Optimizer

The trajectory optimizer utilizes the single time period optimizer to calculate specific economic strategies for each time period. Thus, the single time period optimizer is used once per time period for each iteration of the trajectory optimizer. For example, in a twenty time period calculation which runs for thirty iterations the single time period optimizer will be used 20 x 30 or 600 times.

The single time period optimizer also is an iterative system which uses a lagrange multiplier methodology to calculate an optimal economic strategy for each time period. Given a specific inventory of production capital, a specific set of demand curves for the time period, and a specific estimate of the importance of capital investment, the single time period optimizer will select that specific set of activities (production, investment, and consumption) which would maximize the total utility--assuming that the future value of capital investment has been correctly estimated.

Fundamentally, the single time period optimizer achieves its solution by simulating the operation of the law of supply and demand in an ideal free market. With each iteration there is a refinement in the estimated price or value for each commodity within the system. Given a new estimated set of prices each activity is separately optimized to determine how much it shoudl produce or consume at those prices. The resulting balance of supply and demand is then checked and the prices or values are adjusted to reduce the discrepancy between supply and demand. If the supply is less than the demand for a commodity, the price is raised; if the supply is greater than the demand, the price is lowered. This iterative convergence process stops when differences from iteration to iteration become small enough that they are not likely to distort the overall economic trajectory.

In order to achieve efficient and reliable convergence, the actual computational process within the single time period optimizer is more complicated than is implied by the foregoing basic concept. The actual algorithms that are used are discussed in considerable detail in the second volume, DYNEVAL Documentation.

2.5.5 <u>Calculation of the Horizon Equilibrium</u>

The horizon equilibrium is calculated by an iterative lagrange multiplier algorithm which is very similar to what is used in the single time period optimizer. Specifically, in the equilibrium optimization, each iteration produces a set production and consumption activities that are optimal for the assumed price or value of the commodities. After each iteration, the balance of supply and demand is checked and a refined set of prices are generated which are designed to reduce the imbalance.

The essential differences between the two calculations is that in the single time period optimization capital investment activity can be temporarily very high or very low relative to the total invested capital in each sector, whereas in the horizon equilibrium calculation there is a simple fixed relationship between the capital investment rate and the equilibrium inventory of production capital. (Specifically, the capital investment rate must be exactly equal to the equilibrium inventory multiplied by the sum of the depreciation rate plus the population growth rate).

Whereas, in the single time period optimizer, the capital inventory is fixed, and the rate of capital investment is a free variable to be determined in the optimization; in the horizon equilibrium calculation, the inventory of production capital is a <u>free</u> variable (to be determined by optimization) whose selection <u>implies</u> a specific rate of capital investment.

2.6 INTERPRETATION OF THE SHADOW VALUES

The DYNEVAL output includes a rather comprehensive set of shadow values. To correctly interpret these shadow values it is necessary to understand how both the resource units and the shadow value units are defined within the model.

Within DYNEVAL all resources are measured in monetary value units using the price structure that prevailed in the data base economy. Specifically, if one is using the 1972 Soviet economic data base, then all resources will be measured in terms of their ruble value in the 1972 economy, and in particular, the model will know how many rubles worth of crude oil (in 1972 prices) is needed by various sectors of the economy, but it will not know how many barrels of crude oil that implies. Similarly, the model will know how many rubles worth of rubber products is required by the automobile industry but it will not know how many pounds or how many tires that implies. Thus within the model, physical commodities such as oil, tires, and food products are measured exclusively in terms of their monetary value in the data base economy.

The shadow values for resources, however, are measured in utility units, not rubles or dollars. The utility scale, however, is defined within the model so that one rubles' worth of each commodity in the base year will correspond to one utility unit of value to the economy in the base year. As a consequence of these definitions, the shadow price of all commodities in the data base economy is <u>exactly</u> one utility unit per ruble of commodity value (using 1972 ruble prices). Similarly, if the model were run using the 1967 U.S. economy as the data source, all resources would be measured in terms of dollar value using 1967 U.S. prices. The shadow values within the model would then be in utility units per 1967 dollar's worth of each commodity.

Because the shadow values deal with units of utility, rather than monetary value they do not reflect, and cannot be expected to reflect the effects of monetary inflation or deflation. The changes in shadow values that are seen within the model, therefore, represent the dynamic changes in the real value of specific resources to the economy, rather than changes in the effective value of money. The following sections discuss briefly the specific types of shadow values that are represented within the model.

Value of Each Commodity, λ. In the output of the model specific time dependent values are displayed for each of the comodities. In the data base economy these values, are by definition, equal to 1.0. However, in subsequent years the value can be either higher or lower than 1.0. For example, in a severely damaged post attack economy the utility values of certain critical economic resources can be as much as 100 times greater than it was in the data base economy. On the other hand, if the economy continues to grow and expand beyond the data base economy, most commodities will become more abundant and their unit value to the economy will decrease.

Value of Labor, λ_1. The model calculates a marginal value of labor for each time period. The value of labor in the data base economy is equal to 1.0 by definition, since labor (like the commodities) is measured in monetary value units using the price of the data base economy.

In specific economic scenarios the marginal value of labor can take on values both above and below 1.0. For example, in post attack recovery scenarios where population is evacuated or otherwise protected from heavy losses, there can be a surplus of labor relative to the available production resources in the early recovery phase. During such periods the model will show a relatively low shadow value of labor--reflecting an economic environment in which good jobs are scarce. On the other hand, in a defense mobilization scenario, where it is desired to operate existing production equipment several shifts a day, the model will reflect the labor shortage by placing a higher marginal value of labor--as is appropriate in an economy where good jobs are plentiful.

Value of Production Capital, $\hat{\lambda}$. Logically one can think of two quite different values of production capital. On the one hand there is the investment or purchase cost of new production facilities. On the other hand there is the discounted future value of the production capital. Within the model the value of production capital τ is associated with the discounted future value of the capital. The purchase cost of new capital equipment can be calculated very simply by summing the direct requirements multiplied by the marginal value, λ, of the specified resources. Specifically, the shadow cost of capital equipment i in time period k is given by:

$$\sum_j B_{ij} \lambda_{jk}$$

The final trajectory developed by DYNEVAL, however, is in a dynamic equilibrium so that the discounted future value will usually be almost exactly equal to the production cost for any capital equipment that is actually being produced. There is, however, one exception. Under many scenarios there are certain types of capital equipment which are not produced and should not be produced in specific time periods. For such equipment the discounted future value τ will typically be substantially less than the present capital cost.

Just as with the other shadow values, the shadow value for capital equipment in the data base economy is by definition equal to 1.0. The shadow values in other scenarios can be higher if the particular type of production capital is in short supply, or it can be lower in a mature economy where the supply is more than adequate to meet production requirements.

The model also provides an estimate of the discounted future value of the capital equipment that is available at the beginning of the run. In scenarios involving industrial targeting these shadow values can provide useful information concerning targeting priorities that would most effectively degrade economic performance.

Rental Value of Capital Equipment, U. The "rental value" of capital equipment within a time period represents the marginal value that would accrue to the economy per unit of additional production capital, if the additional capital were available only within the specified time period. Unlike the other shadow values, this rental value is not equal to 1.0 in the data base economy. Obviously, if the purchase cost (and discounted future value) of new production capital is defined to be equal to 1.0 (in the data base economy) the rental value can not also be equal to 1.0. Typically the annual rental value will be somewhere around 10% to 20% of the value or purchase cost. The effective rental value of the capital equipment, like the other shadow values can vary widely depending on the extent to which the specific production equipment is in short supply.

3.0 OVERVIEW OF MATHEMATICAL FORMULATION

This section provides a compact mathematical description of the basic economic equations that are solved by DYNEVAL. To facilitate understanding, the equations are presented in a simplified form, which does not take into account some of the mathematical complications that arise when the specified capital gestation times do not correspond exactly to the time period boundaries. (The reader is warned that the mathematical symbols used in this volume are not consistent with those in Volume II. The notation used here is designed primarily for easy comprehension, whereas the notation in Volume II has been chosen to be as consistent as possible with the actual Fortran code.)

In its present form the model deals with three classes of activities: X_{ik}, Y_{ik}, and Z_{ik}. The level of production in the i<u>th</u> sector of the economy for the k<u>th</u> time period is represented by the variable X_{ik}. The level of capital investment going into the i<u>th</u> sector of the economy for the k<u>th</u> time period is represented by the variable Y_{ik}. The consumption activities (which do not necessarily correspond to specific economic sectors) are represented by the consumption variables Z_{ik}.

The model has been operated using up to 30 production and capital investment activities, 33 consumption activities, and 30 resource constraints for a 40 year recovery trajectory.

The mathematical formulation can be summarized in a simplified form as follows.

DEFINITIONS

R_{jk}	- inventory of resource j at start of year k
K_{ik}	- capital inventory for industry i for year k
$g(i)$	- capital gestation time for industry i
$d(i)$	- depreciation for capital in industry i
$P_i(K, L)$	- production capacity of industry i as a function of capital and labor
X_{ik}	- level of $\underline{ith}$ production activity in year k
α_{ij}	- consumption of resource j per unit activity X_{ik} (negative value implies production)
L_{ik}	- labor allocated to industry i in year k
Y_{ik}	- capital invested in industry i for year k
β_{ij}	- consumption of resource j per unit investment activity i
Z_{ik}	- level of $\underline{ith}$ consumption activity in year k
γ_{ij}	- consumption of resource j per unit consumption activity i
$\mathcal{L}_k$	- leisure time in year k
p	- population growth rate
ϑ_k	- total pool of labor plus leisure time for year k
λ_{jk}	- marginal value of resource j in year k
π_{ik}	- rental value per unit of capital K_{ik} in year k
λ_k^{ℓ}	- marginal value of labor in year k

λ_{ik}	- marginal value of producing capital resource K_{ik} in year k
ρ	- discount rate for future utilities
$\tilde{\rho}_i$	- difference between apparent discount rate for industry i and nominal global discount rate
$U_i(Z_{ik})$	- economic utility for a specified per capita level of the $\underline{ith}$ consumption activity
$D_i(Z_{ik})$	- nonlinear demand functions corresponding to the economic utility functions above
f_i, G_i, B_i	- coefficients of Translog Production Function (Eq. 6)
a_i, b_i	- coefficients of constant elasticity utility function (Eq. 10)

CONSTRAINT EQUATIONS INCLUDED IN DSA ECONOMIC MODEL

Conservation of Resources R_j

$$R_{j,k+1} = R_{j,k} + X_{jk} - \sum_i (\alpha_{ij} X_{ik} + \beta_{ij} Y_{ik} + \gamma_{ij} Z_{ik}) \qquad (1)$$

$$R_{j,k} \geq 0 \text{ for all } j,k \qquad (2)$$

Conservation* of Production Capital K

$$K_{i,k} = [1 - d(i)] K_{i,k-1} + Y_{i,k-g(i)} \qquad (3)$$

where $g(i)$ = capital gestation time

$$K_{i,k} \geq 0 \quad \text{for all } i,k \qquad (4)$$

Production Constraints (Translog Production Function)

$$X_{ik} \leq P_i(K_{ik}, L_{ik}) \qquad (5)$$

Where

$$P_i (K_{ik}, L_{ik}) = G_i \left[f_i K_{ik}^{-B_i} + (1 - f_i) L_{ik}^{-B_i} \right]^{-1/B_i} \qquad (6)$$

Labor Limit Constraints

$\sum_i L_{ik} + \mathcal{L}_k \leq$ [illegible]$_k$ (7)

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*The equation used in the model is somewhat more complicated to allow for non-integer gestation times and to reflect the integrated value over the period of time when new capital actually becomes operational.

DEFINITION OF ECONOMIC OBJECTIVES IN DSA ECONOMIC MODEL

The DSA economic model can be viewed in two mathematically equivalent ways, either as an optimizing model which maximizes a time integrated economic utility function, or as an economic equilibrium model which brings supply and demand into balance over the total trajectory of economic development. Viewed as an optimizing model, the objective function U is given as a function of the consumption activities Z_{ik} and in year k

$$U = \sum_{k} \sum_{i} U_i(Z_{ik}) e^{-\rho k} \tag{8}$$

where the components U_i of the utility function can be defined either as a simple logarithmic utility function

$$U_i(Z_{ik}) = a_i \, \ell n(Z_{ik} - Z_i^0) \tag{9}$$

or in the more general form of a constant elasticity utility function.*

$$U_i(Z_{ik}) = a_i \, \frac{[Z_{ik} - Z_i^0]^{1 - b_i} - 1}{1 - b_i} \tag{10}$$

When viewed as an optimizing program the model simply maximizes the total utility function subject to all of the previously stated constraints.

If we wish to interpret the program as a market equilibrium model, then the utility functions $U_i(Z_{ik})$ can be converted into nonlinear demand functions $D_i(Z_{ik})$ where

*The equations in this section are slightly simpler than in the model, because the model uses a utility function which is defined on a per capita basis to allow for population growth.

$$D_i(Z_{ik}) = \frac{\partial U_i(Z_{ik})}{\partial Z_{ik}} \tag{11}$$

CHARACTERISTICS OF THE SOLUTION

Because the two interpretations are mathematically equivalent the same solution methodology is applicable in either case. The optimum solution involves finding sets of values X_{ik}, Y_{ik}, Z_{ik}, R_{jk}, λ_{jk}, K_{ik}, λ_{ij}, L_{ik}, and $\mathcal{X}_k$ such that the following equilibrium conditions apply.

STATIC EQUILIBRIUM CONDITIONS

Demand Equilibrium (marginal value = marginal cost)

$$D_i(Z_{ik}) = \sum_j \gamma_{ij} \lambda_{jk} \tag{12}$$

Production Equilibrium (marginal net value of product = marginal cost of labor)

$$\left[\lambda_{ik} - \sum_j \alpha_{ij} \lambda_{jk} \right] \frac{\partial P_{ik}(K_{ik}, L_{ik})}{\partial L_{ik}} = \lambda_k^{\ell} \tag{13}$$

Investment Equilibrium (marginal value = marginal cost)

$$\lambda_{ik} = \sum_j \beta_{ij} \lambda_{jk} \tag{14}$$

DYNAMIC EQUILIBRIUM CONDITIONS*

The marginal value of production capital equals discounted value to future

$$\Lambda_{ik'} = \sum_{k'+g(i)}^{k=\infty} e^{-\bar{\rho}_i(k-k')} \mu_{ik} \tag{15}$$

where value of capital to future time periods, μ_{ik}, is defined as the marginal productivity of capital multiplied by the net value of the product.

$$\mu_{ik} = \frac{\partial P_i(K_{ik}, L_{ik})}{\partial K_{ik}} \left\{ \lambda_{ik} - \sum_j \alpha_{ij} \lambda_{jk} \right\} \tag{16}$$

and $\bar{\rho}_i$ is given by

$$\bar{\rho}_i = \rho + \tilde{\rho}_i + p + d_i \tag{17}$$

where p is population growth rate, d_i is the depreciation rate, and $\tilde{\rho}_i$ is the difference between effective discount rate in sector i and ρ, the general discount rate. (Note: $\tilde{\rho}_i \neq 0$ is useful in matching existing investment stracegy in a nonoptimal economy.)

SOLUTION FOR STATIC EQUILIBRIUM

The model has a capability to solve for a static equilibrium in which the values of all variables are constant over time. This capability provides a useful quick analysis approach for many problems. It is also used to provide horizon values for the dynamic optimization.

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*The equations used in the model are somewhat more complicated to allow for noninteger gestation times and to reflect the integrated value over the period of time when new capital actually becomes operational.

USER ESTIMATION OF STRUCTURAL PARAMETERS

Most of the structural parameters for the model are determined by the basic input/output data that defines the rate of consumption and production of resources α_{ij}, β_{ij}, and γ_{ij} for each activity. The user, however, is required to estimate a few additional parameters.

For the <u>production activities</u> the user must provide estimates of the following parameters:

g_i the gestation time for new capital
d_i the physical depreciation rate for old capital

In addition, the user must provide an estimate of the labor vs. capital elasticity in the production function for the industry. To do this the user estimates a maximum factor by which the productivity of capital can be increased in each industry by increasing the use of labor. The program uses this information to estimate the factor B_i which controls the capital elasticity for the industry.

The <u>population assumptions</u> must also be specified. Specifically, the user specifies a population growth rate p and a ratio of the available $\mathcal{L}_k$ to the total labor

$$\sum L_{ik}$$

in the base year for which the data is provided. Typically, the total discretionary leisure time (including weekends and evenings) plus weekly labor is about 80 hrs per week.

ESTIMATION OF INVENTORY OF PRODUCTION CAPITAL

The U.S. input/output data does not specify the accumulated inventory of production capital in each industry. The model, however, provides an automatic estimate of this inventory which takes into account the annual capital investment and the depreciation rate. To make the estimate the user can interpret the base year either as an equilibrium state of the economy relative to population growth, or he can treat it as a non-equilibrium situation. In general, the following factors are taken into account in the estimate of the total inventory of production capital K_i in each industry

Y_{ik} - the annual capital investment in year k for industry i

d_i - the user's estimate of the physical depreciation rate for capital in the industry

p - *the population growth rate*

g_i - the user's estimate of the rate of growth of industry i in excess of the population growth

Because the historical capital investment in any specific base year can be high or low relative to the equilibrium level, the user mav wish to adjust the level Y_{ik} shown in the IO tables for any given base year to correspond more closely to an equilibrium investment level before proceeding with this estimate of the accumulated production capital for the industry.

If other sources of data on the actual production capital are available, the user may wish to iteratively adjust the estimated depreciation rate to obtain a better match to the data.

CALCULATION OF PRODUCTION FUNCTION

The specific production function P_i (K,L) for industry i is calculated using the invested capital K_{ik}, the labor level L_{ik}, and the production level X_{ik} for the industry in the base year together with the user's estimate of the labor capital trade-off. The resulting productivity of capital as a function of labor applied is then calculated and printed out so that adjustments in the assumptions can be made if desired.

USER ESTIMATION OF DEMAND PARAMETERS

The basic demand coefficients a_i for each consumption activity (see Eq. 10) are automatically calibrated by the program, and this process is discussed in the next section. However, the user does have flexibility to adjust the shape of the demand curve if he wishes. Normally, as a default option, the demand elasticity is set to 1.0 and the minimum feasible consumption is set to zero. If the user wishes to experiment with a different demand elasticity $\bar{b}_i$ or a non-zero minimum of consumption Z^0 he can do so by specifying his preferred values for any of the consumption activities.

CALIBRATION OF THE DEMAND FUNCTION

Before the model can be used to project economic behavior, the utility function (or economic demand function) must be calibrated so that it is compatible with the observed behavior of the economy. This calibration of the economic demand function plays a role in the DSA model that is analogous to the determination of historical statistical correlations that is used to calibrate the statistical coefficients in conventional econometric models. Both types of models use information about past behavior to project future behavior. The traditional econometric models assume that statistical correlations in the future economy will be the same as observed in the past. The DSA economic model uses the underlying economic objectives (or economic demand) observed in past behavior as a foundation for predicting future behavior. This does not mean that the model must necessarily assume constant economic objectives. Indeed, one of the most important applications of the model is to project the response of the economy to changes in economic objectives--as for example in a military mobilization study. However, even for such studies it is important to calibrate the economic demand in the model to historical data to provide a baseline for the analysis of changes in economic demand.

In cases where a precise calibration of the economic demand is not critical, the easiest method is simply to interpret the base year input/output data as an equilibrium state, and then calibrate the economic demand function on that basis. (For greater accuracy the base year activity levels can be adjusted to correspond more closely to a long-term equilibrium trend.) If even greater accuracy is required the existing economic data can be interpreted as a non-equilibrium state. However, this approach is more dependent on analyst judgment in the interpretation of the economic data. We will therefore first describe the method of calibration when the basic input/output data (or adjusted data) is interpreted as an equilibrium state of the economy.

The basic input/output data for the economy specifies not only the coefficients α_{ij}, β_{ij}, and γ_{ij} which determine the resources produced or consumed by each activity, but it also specifies the level of operation X_i, Y_i, and Z_i for each activity. If these levels are interpreted as equilibrium levels then the demand function can be easily calibrated. Since the resources consumed and produced in the base year are measured in terms of dollar value in that year the shadow value λ_{jk} for each resource in the base year is simply 1.0 by definition. Using Eq. 12 in which both λ_{jk} and γ_{ij} are known it is therefore possible to calculate the value of the demand functions $D(Z_{ik})$ for the per capital consumption level in the base year k. For any assumed or specified value of demand elasticity b_i and the minimum required activity Z_i^0 the demand coefficient a_i is calculated automatically for each consumption activity.

This calibration of the single year consumption demand, however, is not sufficient to completely determine model behavior. In particular, the rate of capital investment in the model will depend on the discount rate ρ that is applied in evaluating future values. Thus, the discount rate must also be evaluated.

Assuming that the rate of capital investment Y_{ik} observed in the base year k is close to optimum then the marginal value for new capital λ_{ik} for industry i in the base year k can be calculated directly using Eq. 14. Moreover, the rental value of the same capital in the base year is given by Eq. 16. If the base year is interpreted as an economic equilibrium then one can make use of Eq. 15 to calculate an effective discount rate $\bar{\rho}_i$ for the industry. Using Eq. 17 this effective discount rate can be decomposed into the depreciation rate, an average discount rate ρ for the economy as a whole, and an effective discount error term $\tilde{\rho}_i$ for industry i.

If the model is run using these values of ρ and $\tilde{\rho}_i$ it will reproduce the base case economy as a per capita equilibrium state.

The calibration procedures for the model, however, also permit the user to calibrate the demand function under the assumption that the base year is not really an equilibrium economy. To do this, two other parameters are provided which can be adjusted to reflect the user's interpretation of the baseline economy. Specifically, the user can specify for each production activity a physical growth rate g_i for the activity relative to the population growth rate, and a rate of change v_i for the rental value of capital equipment in the industry relative to other prices in the economy. With these assumptions the model will provide a new calibration of the economic demand. If the model is run with such a calibration which assumes a growth in the economy in excess of the population growth rate the model will generate a higher equilibrium and will project an economic growth toward that equilibrium.

In its present form the DSA economic model does not include the effects of technological innovation which increase productivity and produce long-term growth. Consequently, such long-term economic growth does not appear in the model calculations.

SIZE ANALYSIS

For a size comparison with other economic models users are sometimes interested in the total number of equations in the DSA economic model. The actual number of equations depends, of course, on the number of sectors and number of time periods that are selected for any specific calculation. The following analysis sows how the count of equations scales with problem size.

SIZE FACTORS

m - resources

n - production activities

n' - consumption activities

l - time periods

COUNT OF EQUATIONS

m x l - resource conservation

n x l - production K conversation

n x l - production constraints

l - labor constraints

n' x l - utility equations

n' x l - demand equilibrium

n x l - production equilibrium

n x l - investment equilibrium

n x l = dynamic equilibrium

(5n + 2n' + m + 1)l equations total.

For example, in some of our recent use of the model we have used

m = 30 resources

n = 30 production activities

n'= 33 consumption activities

l = 40 time periods

This results in

[(5 x 30) + (2 x 33) + (31)] x 40

[150 + 66 + 31] x 40 = 9880

as the total number of equations.

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